On Multi-Stochastic Tensors and its Applications

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Abstract

In this talk, we presented the Birkhoff-von Neumann type theorem for a multi-stochastic tensors. In particular, we give a necessary and sufficient condition such that a multi-stochastic tensor is a convex combination of finitely many permutation tensors. It is well-known that extreme points in the set of doubly-stochastic matrices are just permutation matrices. However, we find that extreme points in the set of multi-stochastic tensors are not just permutation tensors. Hence some necessary and sufficient conditions for a multi-stochastic tensor to be an extreme point are established. These conditions characterize the “generators” of multi-stochastic tensors. An algorithm to search the convex combination of extreme points for an arbitrary given multi-stochastic tensor is developed. Based on our obtained results, some expression properties for 3rd-order and n-dimensional multi-stochastic tensors (n=3 and 4) are derived, and all the extreme points of the 3-dimensional and 4-dimensional triply-stochastic tensors can be produced in a simple way, respectively. As an application, a new approach for the partially filled square problem under the framework of multi-stochastic tensors is also given.

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