Interval Matrix Eigen/ Singular-Value Decomposition and an Application

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Outline

- Why interval data and interval matrix?
- Eigenvalue/singular values of an interval matrix
- Interval computing
- Practical approach for interval matrix eigenvalue/singular value decomposition
- An application on computational finance
Interval data

- **Source**: Data collected from observation and computing contain error inevitably.
- **Nature**: Variability and uncertainty are the nature of real world phenomena.
- **Ranges vs. Points**: Qualitative indicators are often presented as ranges rather than points.
- **Stream vs. Spot**: Segment and cross intersection.
Interval matrix

- Interval matrix game
- Interval decision making matrix
- Interval coefficient matrix in interval linear systems of equations
- Interval normal matrix in function least-squares approximation
- Interval Jacobean for nonlinear dynamic systems
Interval eigenvalue/vectors

1. Let $A$ be an interval matrix.

2. We call $\Lambda$ and $X$ the eigenvalue and eigenvector sets of $A$. If for any $\lambda \in \Lambda$, $\exists$ a nonzero vector $x \in X$ and $A \in A$ such that $\lambda x = Ax$; and for any nonzero vector $x \in X$, $\exists \lambda \in \Lambda$ and $A \in A$ such that $\lambda x = Ax$. 
Interval singular value

6 Similarly, the singular value set of an interval matrix of $A$ consists of diagonal matrices $\Sigma$, $\exists \forall A \in A$, $\exists$ orthonormal matrices $U$ and $V$, such that $A = U \Sigma V$

6 The challenge is to computationally find the eigenvalue and singular value sets of an interval matrix
Interval computing

- Moore proposed interval computing in later 1950's
- Operations:
  - Arithmetic: +, -, •, ÷
  - Set: ∩, ∪, ¬
  - Logic: <, =, >, ⊂, ⊆, ⊇
  - Utility functions: midpoint(), width(), I/O
- Hardware and software:
  - Intel Itanium processor
  - Sun Studio, C++ standard library, Interval BLAS
- Ups and downs, past and the current
Computational approaches

- Interval representations
  - Endpoint representation
  - Midpoint-radius representation
  - Computer representation
  - Notations
- Practical approach
  - An application
An application in computational finance

- Chen, Roll and Ross stock market forecasting (1986): changes in the stock market \( (SP_t) \) are linearly determined by the following five macroeconomics factors:
  - Growth rate variations of seasonally-adjusted Industrial Production Index (IP),
  - Changes in expected inflation \( (DEI_t) \)
  - Changes in unexpected inflation \( (UI_t) \),
  - Default risk premiums \( (DEF_t) \), and
  - Unexpected changes in interest rates \( (TERM_t) \)
## Point dataset

<table>
<thead>
<tr>
<th>Date</th>
<th>UI</th>
<th>DI</th>
<th>SP</th>
<th>IP</th>
<th>DF</th>
<th>TM</th>
</tr>
</thead>
<tbody>
<tr>
<td>30-Jan</td>
<td>-0.00897673</td>
<td>0</td>
<td>0.014382062</td>
<td>-0.003860512</td>
<td>0.0116</td>
<td>-0.0094</td>
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<td>30-Feb</td>
<td>-0.00671673</td>
<td>-0.0023</td>
<td>0.060760088</td>
<td>-0.015592832</td>
<td>-0.0057</td>
<td>0.0115</td>
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<td>30-Mar</td>
<td>-0.00834673</td>
<td>0.0016</td>
<td>0.037017628</td>
<td>-0.00788855</td>
<td>0.0055</td>
<td>0.0053</td>
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<td>30-Apr</td>
<td>0.00295327</td>
<td>0.0005</td>
<td>0.061557893</td>
<td>-0.015966279</td>
<td>0.01</td>
<td>-0.0051</td>
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<td>30-May</td>
<td>-0.00744673</td>
<td>-0.0014</td>
<td>-0.061557893</td>
<td>-0.028707502</td>
<td>-0.0082</td>
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<tr>
<td>30-Jun</td>
<td>-0.00797673</td>
<td>0.0005</td>
<td>-0.106567965</td>
<td>-0.046763234</td>
<td>0.0059</td>
<td>0.0025</td>
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</tbody>
</table>

...

| 04-Jun | 0.00312327 | -0.0002 | 0.026818986 | 0.005903385 | -0.0028 | 0.0115 |
S & P 500 interval forecasts

Figure 2. Out-of-sample 10-year rolling interval forecasts
## Comparisons

<table>
<thead>
<tr>
<th>Method</th>
<th>Average Error</th>
<th>Standard Deviation</th>
<th>Accuracy Ratio</th>
<th>Total # of Missing</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OLS</strong></td>
<td>0.20572</td>
<td>0.18996</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Confidence Interval based approaches</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>std dev. (95%)</td>
<td>0.723505</td>
<td>0.311973</td>
<td>0.125745</td>
<td>5</td>
</tr>
<tr>
<td>(90%)</td>
<td>0.617097</td>
<td>0.338792</td>
<td>0.145145</td>
<td>7</td>
</tr>
<tr>
<td>std error (95%)</td>
<td>0.36549</td>
<td>0.431112</td>
<td>0.1219</td>
<td>35</td>
</tr>
<tr>
<td>(90%)</td>
<td>0.365712</td>
<td>0.430969</td>
<td>0.104936</td>
<td>36</td>
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<tr>
<td>Low-up bounds</td>
<td>0.066643</td>
<td>0.040998</td>
<td>0.4617</td>
<td>0</td>
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<tr>
<td>Inner Approx.</td>
<td>0.073038</td>
<td>0.038151</td>
<td>0.385531</td>
<td>0</td>
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<tr>
<td><strong>Interval comp.</strong></td>
<td><strong>0.0516624</strong></td>
<td><strong>0.032238</strong></td>
<td><strong>0.641877</strong></td>
<td><strong>0</strong></td>
</tr>
</tbody>
</table>

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Singular value decomposition

Period = 3.0833
Conclusion and acknowledgements

- Initial study on interval matrix singular value decomposition has discovered interesting results
- Further studies are needed

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