Height Functions in Arithmetic Dynamics

Joseph H. Silverman (Brown University)

NCTS — Wednesday 18 October, 2006

The arithmetic dynamics in the title refers to the study of number theoretic questions coming from discrete dynamical systems obtained by iteration of maps on algebraic varieties. An important tool in arithmetic dynamics is the canonical height $\hat{h}_\phi$ associated to a morphism $\phi : \mathbb{P}^N \to \mathbb{P}^N$ of degree $d \geq 2$ and defined by the usual Tate limit

$$\hat{h}_\phi(P) = \lim_{n\to\infty} d^{-n} h\bigl(\phi^{\circ n}(P)\bigr).$$

In this talk I will discuss two questions:
(1) When can two morphisms have the same canonical height?
(2) To what extent does the sup norm

$$\sup_{P\in\mathbb{P}^N} \bigl|\hat{h}_\phi(P) - \hat{h}_\psi(P)\bigr|$$

provide a good measure of the "arithmetic distance" between ϕ and ψ? (Joint work with Shu Kawaguchi)

Independence of Heegner Points on Elliptic Curves

Joseph H. Silverman (Brown University)

NCTS — Thursday 19 October, 2006

Let $E/\mathbb{Q}$ be an elliptic curve and let $\Phi : X_0(N) \to E$ be a modular parametrization of E . Let $k_1, \ldots, k_n$ be distinct quadartic imaginary fields and let $P_1, \ldots, P_n \in E(\bar{\mathbb{Q}})$ be Heegner points attached to the maximal orders of $k_1, \ldots, k_n$. In this talk I will describe these quantities and sketch a proof that if the odd parts of the class numbers of $k_1, \ldots, k_n$ are larger than a certain constant $C(E, \Phi)$, then the points $P_1, \ldots, P_n$ are independent in $E(\bar{\mathbb{Q}})$ modulo torsion. (Joint work with Mike Rosen)

p-adic Dynamics on $\mathbb{P}^n$ and p-adic Green Functions

Joseph H. Silverman (Brown University)

NCTS — Friday 20 October, 2006

The classical Green function associated to a morphism $\phi : \mathbb{CP}^N \to \mathbb{CP}^N$ of degree d is a harmonic function G on $\mathbb{C}^{N+1} \smallsetminus \{0\}$ satisfying the functional equation

$$(G \circ \phi)(\mathbf{z}) = dG(\mathbf{z}).$$

In this talk I will explain how to construct a p-adic analog of the Green function, describe some of its properties, and give applications to local height functions and to p-adic dynamics. (Joint work with Shu Kawaguchi)

Elliptic Divisibility Sequences

Joseph H. Silverman (Brown University)

National Central University — Wednesday 25 October, 2006

Let $E/\mathbb{Q}$ be an elliptic curve and let $P \in E(\mathbb{Q})$ be a point of infinite order. If we write the multiples of P as

$$[n]P = \left(\frac{A_n}{D_n^2}, \frac{B_n}{D_n^3} \right),$$

then the sequence $(D_n)_{n \geq 1}$ is called an *elliptic divisibility sequence*, because it has the property that if $m|n$, then $D_m|D_n$. With an appropriate choice of sign, an elliptic divisibility sequence is a solution to the nonlinear recursion

$$D_{m+n} \cdot D_{m-n} = D_{m+1} \cdot D_{m-1} \cdot D_n^2 - D_{n+1} \cdot D_{n-1} \cdot D_m^2.$$

In this talk I will discuss some of the arithmetic properties of elliptic divisibility sequences, including their rate of growth, variation of sign, properties modulo prime powers, and conjectures related the greatest common divisor of two such sequences.