Rapid Fluctuations of Chaotic Maps on $\mathbb{R}^N$

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The iterates $f^n$ of a chaotic map $f$ display heightened oscillations (or, fluctuations) as $n \to \infty$. If $f$ is a chaotic interval map in one-dimension, then it is now known that the total variation of $f^n$ on that interval grows exponentially with respect to $n$ [15]. However, the characterization of chaotic behavior of maps in multidimensional spaces is generally much more challenging. Here, we generalize the definition of bounded variations for vector-valued maps in terms of the Hausdorff measure and then use it to study what we call rapid fluctuations on fractal sets in multidimensional chaotic discrete dynamical systems. The relations among rapid fluctuations, strict turbulence and positive entropy are established for Lipschitz continuous systems on general $N$–dimensional Euclidean spaces. Applications to planar monotone or competitive systems, and triangular systems on the square are also given. This talk is based on a joint paper by Y. Huang, G. Chen and D. Ma.